OPTICAL PROCESSES IN NANOSTRUCTURED SEMICONDUCTORS
We present a possibility to model morphology of surfaces of nanostructured semiconductor films by use of methods of nonlinear physics. We also describe experimental photoluminescence and absorption spectra in nanostructured fractal medium. By use of our theory we describe the new experimental fact of light localization in non-crystalline films. We suggest a fractal model for energy spectra of excitons in nanostructured semiconductors with porous structure. We obtain equations for exciton and multi-exciton energy depending on energy of exciting photon.
Fractal properties of self-similar and self-affine sets

\[ D = d + \gamma, \quad 0 \leq d \leq n, \quad 0 < \gamma < 1. \]  
(1)

\[ D \] – fractal dimension, \( d \) – topological dimension

\[ P(I) = e^{-l}, \quad P(I) = \int_{I}^{\infty} f(I) dI, \quad f(I) = P(I) = e^{-l}, \quad \int_{0}^{\infty} f(I) dI = 1, \]  
(2)

\[ S(I) = \int_{I}^{\infty} f(I) dI = (I + 1) e^{-l}, \quad 0 \leq S(I) \leq 1. \]  
(3)

\( I \) – quantity of information, \( P(I) \) – probability function,

\( f(I) \) – density of probability distribution function, \( S(I) \) – informational entropy

\[ f(I) = I, \quad e^{-l} = I, \quad I = I_1 = 0.567, \]  
(4)

\[ S(I) = I, \quad (1 + I) e^{-l} = I, \quad I = I_2 = 0.806. \]  
(5)

Transition from self-similarity to self-affinity: \[ e^{-l} \approx 1 - I, \quad \Rightarrow \quad I_{20} = 0.618. \]

Numbers \( I_1, I_2 \) are analogue \( I_{20} \) which is «gold mean» of dynamical measure for self-affine and self-similar systems with informational and statistical properties.

\[ D = \tilde{d} - S, \]  
(6)

\( \tilde{d} \) – topological dimension of the space where an object is embedded

\[ \gamma = \tilde{d} - d - S, \quad 1 - S \leq \gamma \leq \tilde{d} - S, \quad S = (I_1, I_2), \quad \tilde{d} = 2, 3. \]  
(7)
Modelling of morphology of surfaces of nanostructured semiconductor films

\[ Y_{i+1} = \left( Y_i + \text{sign}(\psi_Y) \right) |\psi_Y| \left( n_i, a_i, p_i, i \right)^2 \left( \frac{Y_i}{Y_0} \right)^{\frac{1}{\gamma_Y}} , \quad Y = (n, a, p) \]  

(8)

\( n, p, a \) – concentration of quasi-particles such as electrons, holes and clusters (defects of different types) in semiconductors,

\( \gamma_Y \) – fractal dimensions of sets with self-similar and self-affine properties,

\( Y_0 \) – equilibrium concentrations of electrons, holes and impurities,

\( \psi_Y \) – wave function

Approximation of closely coupled interface of electrons to clusters

\[ \psi_k (\vec{r}) = \sum_{\vec{R}} \varphi (\vec{r} - \vec{R}) \exp(i k \vec{R}). \]  

(9)

\( \varphi (\vec{r} - \vec{R}) \) – Vanyi function

In the case of loosely coupled interface of electron to cluster:

\[ \psi_n (x) = \psi_p (x) = \psi_a (x) = \cos(k_n x) \cos(k_p x) \cos(k_a x). \]  

(10)
Results of the numerical analyses and experimental data

Experimental data

Theoretical results

Microscopy image for the Ag adsorption on a Si surface.

\[ \gamma_n = \gamma_p = \gamma_a = I_2, \quad n_0 = p_0 = a_0 = 5, \quad n_1 = p_1 = 1, \quad a_1 = 100. \]

Image taken after deposition of In on a Si surface.

\[ n_1 = p_1 = 1, \quad a_1 = 100, \quad n_0 = p_0 = a_0 = 2.6, \quad \gamma_n = 0.53, \quad \gamma_p = 2 + I_2, \quad \gamma_a = 3 + I_2. \]

Results of the numerical analyses and experimental data

**Experimental data**  **Theoretical results**

Images of GaAs surfaces.

(d) \( \gamma_a = \gamma_p = \gamma_n = 1 + I_2 \),
\[ n_1 = p_1 = a_1 = 1, \]
\[ n_0 = a_0 = p_0 = 20, \]

(e) \( \gamma_a = \gamma_p = \gamma_n = I_2 \),
\[ n_1 = p_1 = 1, a_1 = 20, \]
\[ n_0 = p_0 = a_0 = 2, \]

(f) \( \gamma_a = \gamma_p = \gamma_n = I_1 \),
\[ n_1 = p_1 = 1, a_1 = 20, \]
\[ n_0 = p_0 = a_0 = 2. \]

Безрядин Н. Н., Котов Г. И., Арсентьев И. Н., Стародубцев А. А.
Формирование наноструктур в системе Ga_2Se_3 / GaAs //
Физика и техника полупроводников, 2005, вып.9, с. 1025-1028
Results of the numerical analyses and experimental data

**experimental data**

**theoretical results**

Morphology of a surface of GaAs.

\[ \gamma_n = I_2, \gamma_p = \gamma_a = 1 + I_1, n_0 = p_0 = a_0 = 1.5, n_1 = p_1 = 1, a_1 = 15. \]
Results of the numerical analyses and experimental data

**Experimental data**

**Theoretical results**

Image of a surface of $\text{CdTe}$

$$\gamma_n = 3 + I_2, \quad n_1 = p = 1, \quad a = 10, \quad n_0 = 3.$$
Results of the numerical analyses and experimental data

Images of films of ZnO on sapphire and silicon surfaces
(d): $\gamma_n = \gamma_p = 1 + I_{10}$, $a=2$;
(e): $\gamma_n = \gamma_p = 2 + I_{10}$, $a=0.5$;
(f): $\gamma_n = 1 + I_2$, $\gamma_p = I_2$, $a=0.5$.

Жерихин А.Н., Худобенко А.И., Вилльямс Р.Т., Вилкинсон Дж., Усер К.Б., Хионг Г., Воронов В.В. Лазерное напыление пленок ZnO на кремниевые и сапфировые подложки // Квантовая электроника, 2003, №11, с.975-980
Results of the numerical analyses and experimental data

**Experimental data**

**Theoretical results**

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**Images of a surface of GaAs**

(c) \( n_1 = p_1 = a_1 = 1, \gamma_n = I_1, \gamma_p = 2 + I_2, \gamma_a = 3 + I_2, n_0 = p_0 = a_0 = 2.5 \),

(d) \( n_1 = p_1 = 1, a_1 = 50, \gamma_n = \gamma_p = \gamma_a = 1 + I_2, n_0 = p_0 = a_0 = 2.5 \).
**Results of the numerical analyses and experimental data**

**Experimental data**

**Theoretical results**

Morphology of a film of $Si_{1-x}Ge_x$ on a silicon surface.

$$n_1 = p_1 = a_1 = 1, \gamma_n = \gamma_p = \gamma_a = 3 + I_2, n_0 = p_0 = a_0 = 1.$$
Results of the numerical analyses and experimental data

Experimental data  Theoretical results

Image of a film of InGaN on a GaN surface.

\[ n_1 = p_1 = 1, a_1 = 5, \]
\[ \gamma_n = \gamma_p = \gamma_a = I_2, \]
\[ n_0 = p_0 = a_0 = 5. \]

Results of the numerical analyses and experimental data

Morphology of a surface of $\text{SnO}_2$

$$n_1 = p_1 = a_1 = 1, \quad \gamma_n = \gamma_p = \gamma_a = 1 + I_1, \quad n_0 = p_0 = a_0 = 15.$$
Fluctuation-dissipation relation for the description of photon absorption

Absorption coefficient:

\[
\alpha(\omega, E_g) = \alpha_0(E_g) \left[ \frac{1}{\hbar\omega - E_g} \right] \left[ \frac{\hbar\omega - E_g}{2kT} \right] E_e(k) (\hbar\omega - E_g)^{1+\gamma/2}
\]

(11)

\(\alpha_0(E_g)\) – dimensional constant given for the absorption edge

\(E_e(k)\) – power spectra of electron oscillations depending on wave number

\(E_g\) – energy gap

\[
Y_{i+1} = \left( Y_i + \text{sign}(\psi_Y) \right) \left| \psi_Y(n_i, a_i, p_i, i) \right|^2 \left| \frac{Y_i}{Y_0} \right| \left[ \frac{1}{Y} \right] \left( n, a, p \right)
\]

\(C_{e,e}(m) = \lim_{N \to \infty} \frac{1}{N} \sum_{i=0}^{N} n_{e,i+m} n_{e,i}, \quad E_e(k) = \lim_{N \to \infty} \frac{1}{N} \sum_{m=0}^{N} \cos(2\pi mk)C_{e,e}(m)
\]

(12)

\(C_{e,e}(m)\) – spatial correlations of electron density

\(E_e(k)\) – power spectra
Results of the numerical analyses and experimental data

**Experimental data**

**Theoretical results**

Photoluminescence spectrum of InP/GaAs quantum dots at laser irradiation

\[ E_g = 1.27 \text{ eV}, kT = 0.01 \text{ eV}, n_0 = p_0 = a_0 = 1, \gamma_n = \gamma_p = \gamma_a = I_2, n_1 = p_1 = 1, a_1 = 4, \ h\nu_f = 1.33 \text{ eV}. \]

\[- \hbar\nu_f = 1.355 \text{ eV}, \quad - \hbar\nu_f = 1.38 \text{ eV}, \]

\[- \hbar\nu_f = 1.395 \text{ eV}, \quad - \hbar\nu_f = 1.41 \text{ eV}. \]
Photoluminescence spectrum of InGaAs/GaAs quantum dots.

\[ E_{p_0} = 1.245 \text{ eV}, \quad E_g = 1.1 \text{ eV}, \quad kT = 0.0005 \text{ eV}, \]

\[ \gamma_n = \gamma_p = \gamma_a = I_2, \quad n_1 = p_1 = 1, \quad a_1 = 2, \quad n_0 = p_0 = 1, \quad a_0 = 2. \]
Photoluminescence spectrum of CdS nanoparticles

\[ E_g = 1.4 \text{ eV}, \quad \hbar \omega_f = 1.9 \text{ eV}, \quad kT = 0.035 \text{ eV}, \]
\[ n_0 = p_0 = a_0 = 1, \quad \gamma_n = \gamma_p = \gamma_a = I_2, \quad n_1 = p_1 = 1, \quad a_1 = 8. \]

Absorption spectrum of $C_{60}Cl_{24}$

$E_g = 1.6\, eV$, $kT = 0.001\, eV$, $n_0 = p_0 = a_0 = 1$, $\gamma_n = \gamma_p = \gamma_a = I_2$, $n_1 = p_1 = a_1 = 1$,

$1 - \hbar \omega_f = 1.2\, eV$, $2 - \hbar \omega_f = 1.6\, eV$. 

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Разбирин Б.С., Старухин А.Н., Чугреев А.В., Згода А.С., Смирнов В.П., Грушко Ю.С., Колесник С.Г., Коэр П.-Ф., Льевэн Ж., Колэн Р. Линейчатый спектр поглощения галогенофуллерена $C_{60}Cl_{24}$ // Физика твердого тела, 2002, том 44, вып. 11, с. 2106-2111
Photoluminescence spectrum of ZnTe monocrystal

\[ E_g = 2.37 \text{ eV}, \hbar w_f = 2.37 \text{ eV}, kT = 0.002 \text{ эВ}, \ n_0 = p_0 = a_0 = 10, \ \gamma_n = \gamma_p = \gamma_a = I_2 \]


